# Triviality All Ways

Conditionals in Decision Theory
Lecture 5

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# Recap

# Where we are

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But they implement that idea in different formal languages.

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- Skyrmsian CDT (a version we haven't seen before)

$$U_{\mathrm{SK}}(O) = \sum_i \sum_w p(w) \cdot ch_{H_{w,t^-}}(S_i \mid O) \cdot u(O \cap S_i)$$

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Today's question: can the Stalnakerian and Skyrmsian formulations really be equivalent?

# The bridge needed for equivalence

To identify Stalnakerian CDT with Skyrmsian CDT, we need a bridge between:

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That bridge is precisely the kind of principle we are about to discuss:

**Skyrms-style bridge**

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The worry is that this bridge does not vindicate CDT. It threatens to make CDT collapse into ordinary Savage-style expected utility theory.

# Triviality for counterfactuals

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# Counterfactuals and probability

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- What about counterfactuals?

Two problems (corresponding to two different clusters of literature).

i. Assigning intuitive probabilities to counterfactuals.

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ii. Triviality rearing its ugly head.

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What do we mean by chance?

- Roughly: a prominent ‘objective’ interpretation of probability, which is supposed to figure in laws of nature.
- Work in epistemology and metaphysics focuses on capturing the link between chance and credence. Williams’ proof exploits this bridge.

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- The object argument is the proposition that is assigned a chance.
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$$ch_{H_{w,t}}(\mathrm{A})$$

The overall strategy.

i. Start from a counterfactual counterpart of the Thesis (i.e. a principle about credence).
ii. Derive a Thesis-like principle about chance.
iii. Motivate a closure assumption about chance.
iv. Run a Lewis-like triviality result using (i) and (ii).

Skyrms' Thesis.

Let $ch_{H_{w,t}}$ be the chance function at $w$ and $t$. For all A, C, and for all rational credence functions $cr_{E_t}$ such that $E$ is the subject's total evidence at $t$:

$$cr_{E_t}(\mathrm{A} \mathbin{\Box\!\!\rightarrow} \mathrm{C}) = \sum_{w \in W} cr_E(w) \times ch_{H_{w,t-}}(\mathrm{C} \mid \mathrm{A})$$

- Rough gloss: your (counterfactual) suppositional credence at $t$ in C, given A, should equal to your expectation of the conditional chance of $C$, given $A$, at a time that obtained 'just before' $t$.
- Your expectation is determined by your credences in candidates for the chance function in different worlds.

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Notice: Skyrms' Thesis is not the only principle cashing out subjunctive suppositional credences—more on this in a bit!

A bridge principle between credence and chance:

Principal Principle. (Lewis 1980)
$Pr(A|Ch(A) = x) = x$, assuming that $Pr(Ch(A) = x) > 0$

- This says: conditional on the chance of A being $x$, your credence in A should be $x$.
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Example: suppose you're certain that the chance of Spain winning the WC is .3. Then your credence that Spain wins should be .3.

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Does this remind you of something?

One more principle: closure.

**Closure**

If $ch$ is the chance distribution at some world $w$ at time $t$, $\mathbf{A}$ any proposition (such that $ch(\mathbf{A}) > 0$), and $ch_0$ is the probability assignment that results from conditionalizing $ch$ on $\mathbf{A}$ (i.e. $ch_0(\cdot) = ch(\cdot\,|\mathbf{A})$), then there's a world $w_0$ and time $t_0$ whose chances at $t_0$ are described by $ch_0$.

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Pet peeves:

- I suspect that closure is more demanding than Williams thinks!
- By conditionalizing on some propositions, you might land on a probability function that is not the chance function of any world.
- But I also think in the end these problems are not fatal.

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v. $1 \times ch_{H_{w,t}}(C) + 0 \times ch_{H_{w,t}}(\neg C) =$

vi. $ch_{H_{w,t}}(C)$

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- This step just uses the so-called Law of Total Probability.
- Essentially, we just use math and the definition of conditional probability.

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- This step is justified by the idea that chances 'evolve by conditionalization'.
- When we conditionalize a chance function on a proposition, that is the same thing as conjoing that proposition to the grounding argument.

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- This is where we apply the Chancy Equation.
- Notice: we replace chances of conditionals with conditional chances.

iv. $ch_{H_{w,t^-}+C}(C \mid A) \times ch_{H_{w,t}}(C) + ch_{H_{w,t^-}+\bar{C}}(C \mid A) \times ch_{H_{w,t}}(\neg C) =$

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- If history includes the information that C, the chance of C is one.
- Similarly, *mutatis mutandis*, for ¬C.

How to respond?

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(iii) Reject Skyrms' Thesis, and possibly look for an alternative.

A way of going the way of (iii): Santorio 2025.

There I propose to replace the Chancy Equation with the following:

**Counterfactual Chance Thesis**

$ch_{H_{w,t}}(\mathsf{A} \mathrel{\Box\!\!\rightarrow}_s \mathsf{C}) = ch_{s(\mathsf{A},H_{w,t})}(\mathsf{C})$

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$ch_{H_{w,t}}(\mathsf{A} \mathrel{\Box\!\!\rightarrow_s} \mathsf{C}) = ch_{s(\mathsf{A}, H_{w,t})}(\mathsf{C})$

- This says: the chance that $\mathsf{A} \mathrel{\Box\!\!\rightarrow_s} \mathsf{C}$ equals the chance of C, in the closest world history where A is the case.
- Crucially, we use the same selection function for the counterfactual and for the grounding argument of the chance function.
- In a slogan: chances of counterfactuals are counterfactual chances.

## The Counterfactual Chance Thesis

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In Williams' proof, the triviality-generating step is the equality in (4):

(4) $ch_{T_w H_{w,t}+\mathsf{C}}(\mathsf{A} \,\Box\!\!\rightarrow \mathsf{C}) = ch_{T_w H_{w,t}+\mathsf{C}}(\mathsf{C} \mid \mathsf{A})$

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If we switch to CCT, (4) is replaced by:

(4) $ch_{T_w H_{w,t} + \mathsf{C}}(\mathsf{A} \Box\!\!\rightarrow \mathsf{C}) = ch_{T_w s(\mathsf{A}, H_{w,t} + \mathsf{C})}(\mathsf{C})$

- Crucially, the right-hand side of 4 need *not* be identical to 1.
- The reason is that, even if we start from a history that vindicates C, the selection function can take us to a history where C is not true/not settled.

# The Counterfactual Chance Thesis

For illustration, consider this scenario:

Two coins. Sarah is going to flip one of two coins A and B, picked at random. Coin A is fair. Coin B has a 100% bias towards heads.

- Let 'A-Flip' stand for the proposition that coin A is flipped, and 'Heads' be the proposition that whichever coin is flipped lands heads.
- Now, consider the chance of the counterfactual A-Flip $\Box\!\!\rightarrow$ Heads, under the hypothesis that the flipped coin lands heads.

(5) $\quad ch_{H_{w,t}+\textbf{Heads}}(\text{A-Flip} \Box\!\!\rightarrow \text{Heads})$

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- Via the Chancy Equation, this is 1.
- Intuitively, ... it is less than 1.
- The CCT allows it to be less than 1.

(5) $ch_{H_{w,t}+\textbf{Heads}}(\text{A-Flip} \mathrel{\Box\!\!\rightarrow} \text{Heads})$

Given the CCT, (5) equals:

(6) $ch_{s(\textbf{A-Flip},H_{w,t}+\textbf{Heads})}(\text{Heads})$

- Crucially, $s(\text{A-Flip}, H_{w,t} + \text{Heads})$ need not be a scenario that makes heads true.
- In fact, on a plausible choice of selection function, some closest worlds where A is flipped will lead to a tails outcome.
- Hence the chance of heads will be less than 1, hence the chance of A-Flip $\Box\!\!\rightarrow$ Heads, under the hypothesis that the flipped coin lands heads, is less than 1.

## The Counterfactual Chance Thesis

So: we have a recipe to block triviality!

But there is work to do:

- No tenability result for the CCT (yet).
- No idea how it interacts with decision theory.

# Causal decision theory and triviality

## Three versions of CDT

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- Skyrmsian CDT

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> *Several versions of causal decision theory are on the market... But also I shall suggest that we causal decision theorists share one common idea, and differ mainly on matters of emphasis and formulation. The situation is not the chaos of disparate approaches that it may seem. (Lewis, 1981, p. 5)*

# Equivalence?

Here's an even more explicit statement of the equivalence claim, from Joyce (1999):

> *From a purely decision-theoretic perspective, the similarities among the "causally relevant factor," "conditional chance," and "counterfactual dependence" versions of [CDT] are more striking than their differences. A person's judgment about which of the three interpretations is correct will depend on her views about the metaphysics and epistemology of causation rather than her intuitions about how to make choices.... Skyrms and Harper have expressed the same sentiment, observing, "**It can be argued that the various forms of causal decision theory are equivalent**—that an adequate version of any one of [them] will be interdefinable with adequate versions of the others." I think this is basically right; **there is no great difference between any of the approaches we have discussed**. (p. 171)*

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So, if the result holds, antecedents make no difference to the chances of their consequents.

## Apply this to options and states

Now let the antecedent be an option $O$, and let the consequent be a state $S_i$.

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By triviality, this becomes:

$$U_{\mathrm{SK}}(O) = \sum_i \sum_w p(w) \cdot ch_{H_{w,t^-}}(S_i) \cdot u(O \cap S_i).$$

If your credences defer to the relevant chances, then:

$$\sum_{w} p(w) \cdot ch_{H_{w,t-}}(S_i) = p(S_i).$$

# Collapse into Savage

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**Upshot**

If the triviality result for Skyrms's thesis holds, then the bridge CDT'ers need does not vindicate CDT. It makes CDT collapse into Savage!

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If Stalnakerian CDT is equivalent to Skyrmsian CDT, and Skyrmsian CDT collapses into Savage, then Stalnakerian CDT collapses too.

# The moral

The problem is not a new derivation.

It is an application of the triviality result for Skyrms's thesis to decision theory.

If Stalnakerian CDT is equivalent to Skyrmsian CDT, and Skyrmsian CDT collapses into Savage, then Stalnakerian CDT collapses too.

So the equivalence claim is costly. Something has to give:

- the bridge from counterfactuals to conditional chances;
- the Principal Principle;
- closure for chance functions;
- or the claim that these CDT formulations are equivalent.

# Bonus: Desire-as-Belief

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This view has played an important role in ethics (Parfit, 1984).

We are going to show, however, that in the context of decision theory, it faces stark challenges.

# Evidential decision theory, reconsidered

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We have argued that evidential decision theory isn't the correct decision theory.

There is, however, a sense in which it's still useful...

To see this, consider this quote from Lewis (1996):

## Value vs choiceworthiness

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> *Suppose a certain action would serve as an effective means to your ends. Yet at the same time it would constitute evidence—evidence available to you in no other way that you are predestined inescapably to some dreadful misfortune. Should you perform that action?—Yes; your destiny is not a consideration,since that is outside your control. Do you desire to perform it?—No; you want good news, not bad. Since our topic here is not choiceworthiness but desire, and since the two diverge, we adopt an "evidential" conception of expected value, on which the value of the useful action that brings bad news is low. Choiceworthiness is governed by a different, "causal", conception of expected value.*

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Lewis is here contrasting evidential and causal decision theory.

He is saying that EDT is not the right theory of how you should *choose*; but it may still be the right theory of rational subjective value.

# How to read the equation

To make this explicit, let us write $V(A)$ for the evidential expected utility of $A$, considered as the *rational subjective value* you should attach to $A$. In the language of worlds, we have:

$$V(A) = \sum_w p(w \mid A) \cdot u(w)$$

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When $A$ is an available option, $O$, maximizing this quantity is just evidential decision theory.

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Then our equation for evidential expected utility yields:

$$\begin{aligned} V(A) &= p(G \mid A) \cdot V(G) + p(\neg G \mid A) \cdot V(\neg G) \\ &= p(G \mid A) \cdot 1 + p(\neg G \mid A) \cdot 0 \\ &= p(G \mid A). \end{aligned}$$

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So far, so good...

# Desire-as-Belief

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Then Lewis's Desire-as-Belief thesis says (Lewis, 1988, 1996):

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In words: Your degree of desire for $A$ should match your degree of belief that $A$'s truth would be *good*.

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For realists about value, this is a natural thought:

- there are facts about what is good;
- rational belief should track those facts;
- rational desire should track one's beliefs about those facts.

So DAB can look like a value-theoretic analogue of the Principal Principle:

subjective attitudes should match beliefs about objective magnitudes.

# Lewis's ingredients

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The question is: can all three hold generally?

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But, in general, $A$ and $A^\circ$ need not be probabilistically independent.

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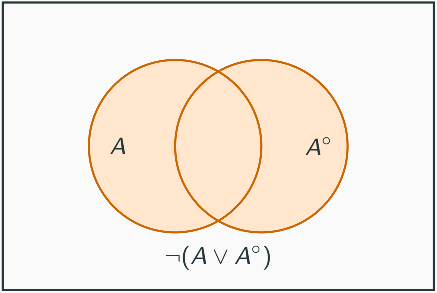

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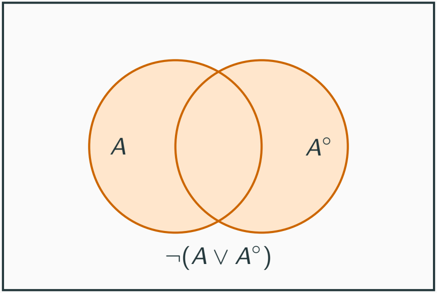


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# Why this is a problem

Suppose initially $A$ and $A^\circ$ are independent:

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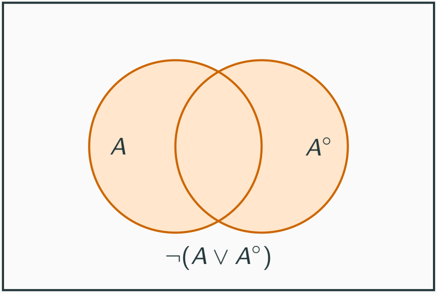


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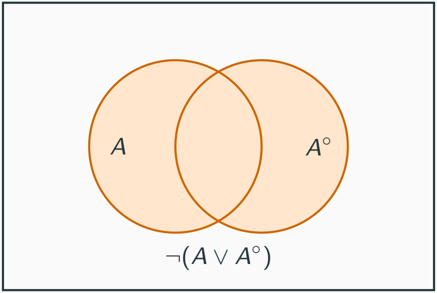


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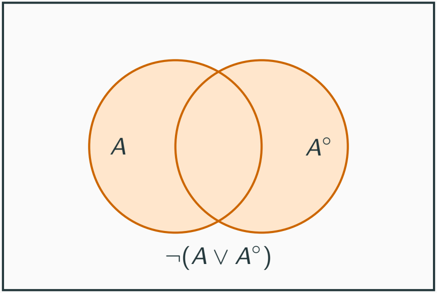


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- $p(A^\circ)$ increases.

So DAB can go from satisfied to unsatisfied.

# First moral

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The problem is that, if we combine DAB with Conditionalization and Invariance, then DAB imposes an extremely strong constraint:

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for arbitrary $A$.

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The problem is that, if we combine DAB with Conditionalization and Invariance, then DAB imposes an extremely strong constraint:

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for arbitrary $A$.

That is the beginning of a triviality result: the thesis can hold only in highly special cases.

# Triviality

# Stalnaker's Thesis

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Example:

*If the die didn't land on* 1*, it landed on* 2*.*

For a fair die, the natural credence is:

$$p(2 \mid \neg 1) = \frac{1}{5}.$$

We heard yesterday, however, that if Stalnaker's Thesis is tru—and given some other plausible assumptions—then

$$p(B \mid A) = p(B).$$

That looks quite a bit like the conclusion of our argument against the Desire-as-Belief Thesis.

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So, if the derivation is sound, then Stalnaker's Thesis can hold generally only in trivial probability spaces.

This is why Lewis's result is not just a problem about one conditional. It is a problem about the whole project of identifying:

probability of a conditional with conditional probability.

# The connection

# DAB as a conditional thesis

Those remarks give us a hint about how we can make the connection between the DAB argument, and the triviality result for Stalnaker's Thesis even closer.

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Let $G$ be the proposition that things are good. Then DAB can be written:

**Conditional DAB**

$$V(A) = p(A \to G).$$

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Let $G$ be the proposition that things are good. Then DAB can be written:

**Conditional DAB**

$$V(A) = p(A \to G).$$

In words:

Your desire for $A$ should match your credence in
*If $A$, then things are/would be good.*

# Jeffrey plus DAB yields Stalnaker

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Putting them together:

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$$p(A \to G) = p(G \mid A).$$

But this is just Stalnaker's Thesis, with $G$ as consequent.

# The shared structure

| | DAB | Stalnaker |
|---|---|---|
| Thesis | $V(A) = p(A^\circ)$ | $p(A \rightarrow B) = p(B \mid A)$ |
| Problematic move | $A^\circ$ is fixed across learning | $A \rightarrow B$ is fixed across learning |
| Result | $p(A^\circ) = p(A^\circ \mid A)$ | $p(B) = p(B \mid A)$ |

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